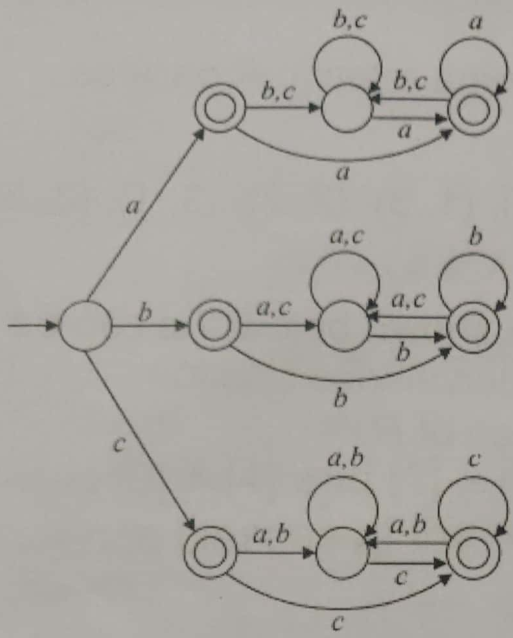
Solution 1:



Solution: 2

~~{a, b, c, abca~~

{ a, abca, abcaa, aa, aba, aca

b, bacb, bacbb, bb

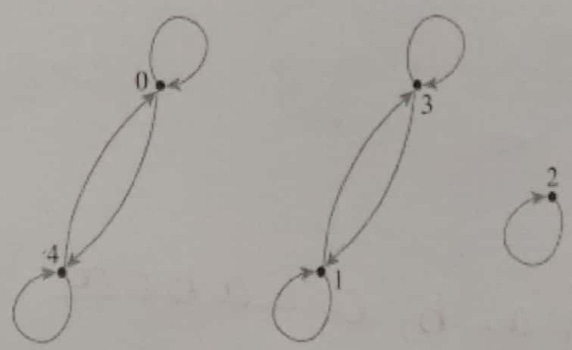
c, cabc, cabcc, cc }

Example 5 – *Equivalence Classes of a Relation Given as a set of Ordered Pairs*

Let $A = \{0, 1, 2, 3, 4\}$ and define a relation R on A as follows:

$$R = \{(0, 0), (0, 4), (1, 1), (1, 3), (2, 2), (3, 1), (3, 3), (4, 0), (4, 4)\}.$$

The directed graph for R is as shown below. As can be seen by inspection, R is an equivalence relation on A. Find the distinct equivalence classes of R.



First find the equivalence class of every element of A.

$$[0] = \{x \in A \mid x\ R\ 0\} = \{0, 4\}$$

$$[1] = \{x \in A \mid x\ R\ 1\} = \{1, 3\}$$

$$[2] = \{x \in A \mid x\ R\ 2\} = \{2\}$$

$$[3] = \{x \in A \mid x\ R\ 3\} = \{1, 3\}$$

$$[4] = \{x \in A \mid x\ R\ 4\} = \{0, 4\}$$

Note that [0] = [4] and [1] = [3]. Thus the *distinct* equivalence classes of the relation are

{0, 4}, {1, 3}, and {2}.

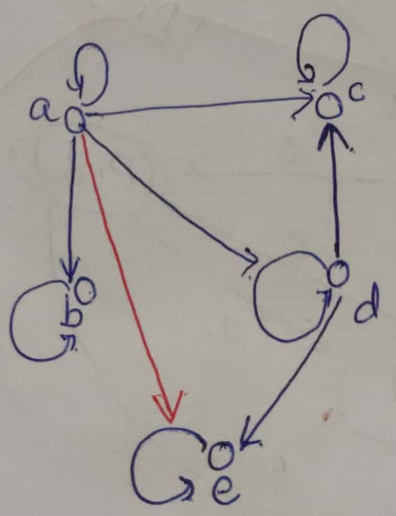
Relation R1 = {(0,0), (1,1), (2,2), (3,3), (4,4) (0,4), (1,3)} which satisfy the property of reflexivity, transitivity, antisymmetric.

R1 ={{(0,0), (1,1), (2,2), (3,3), (4,4), (4,0), (3,1)}

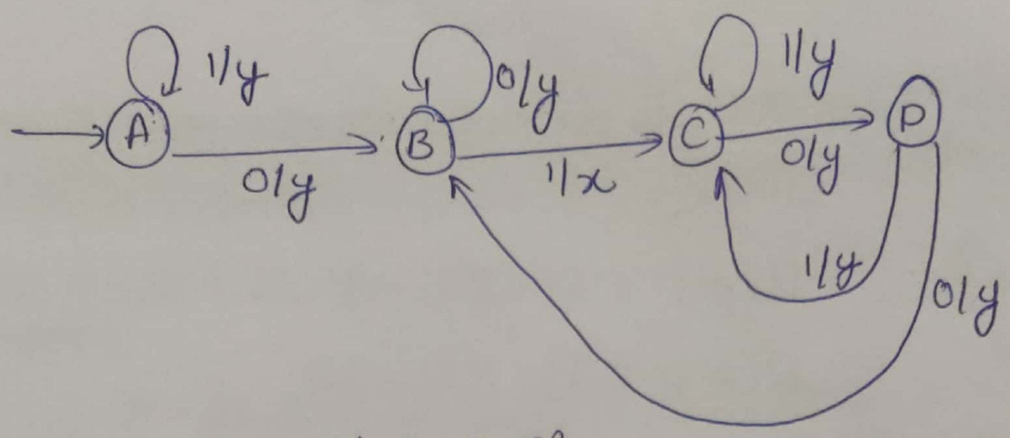
Q.3) R ={(a, b), (a, c), (a, d), (d, c), (d, e)}

R* = {(a, a), (b, b) (c, c) (d, d) (e, e), (a, b), (a, c), (a, d), (d, c), (d, e)}

Directed graph: (a, e)



Q4.



no. of states = {A, B, C}

no. of input symbol = {0, 1}

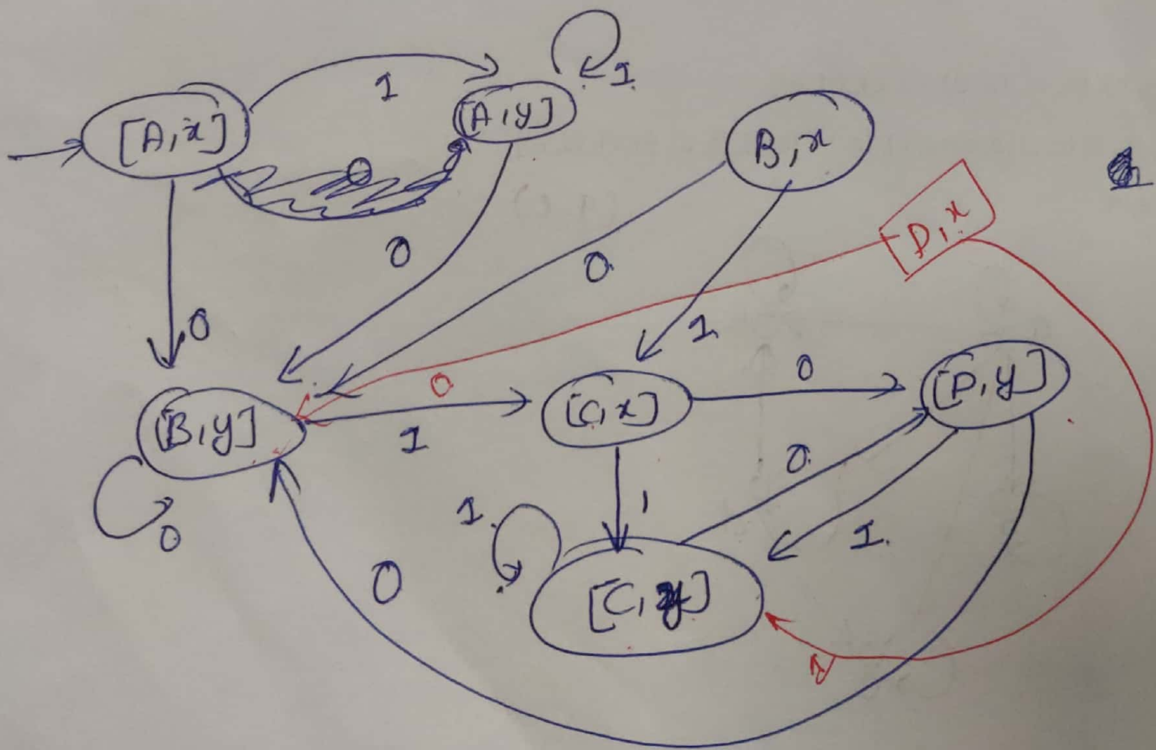
no. of o/p symbol = {x, y}

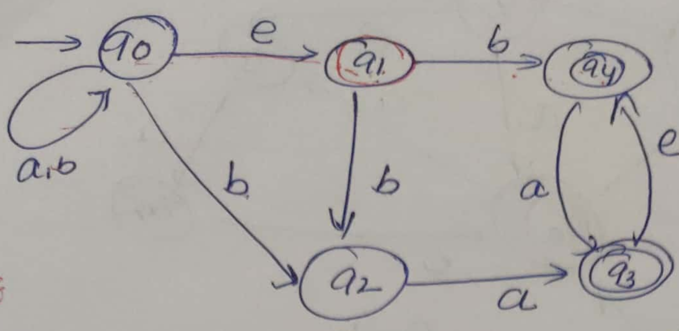
Input: 10011010

O/p: y y y x y y y y

no. of state × o/p symbol =

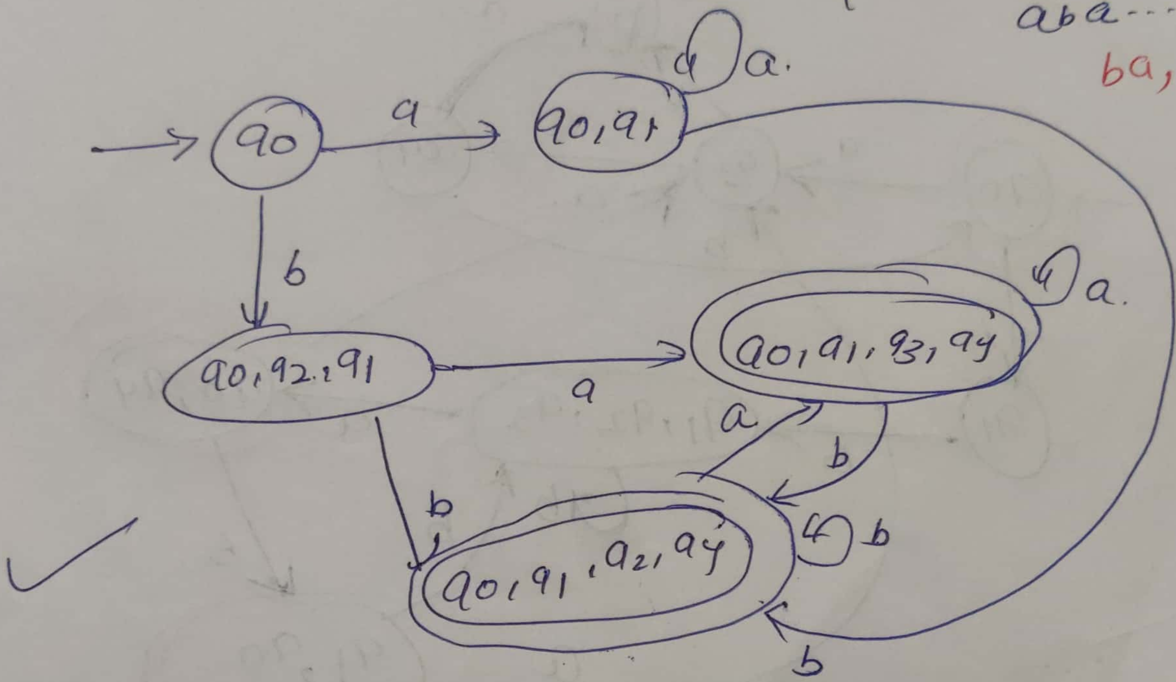
{A, B, C} {x, y} = {(A, x) (A, y) (B, x) (B, y) (C, x) (C, y)}

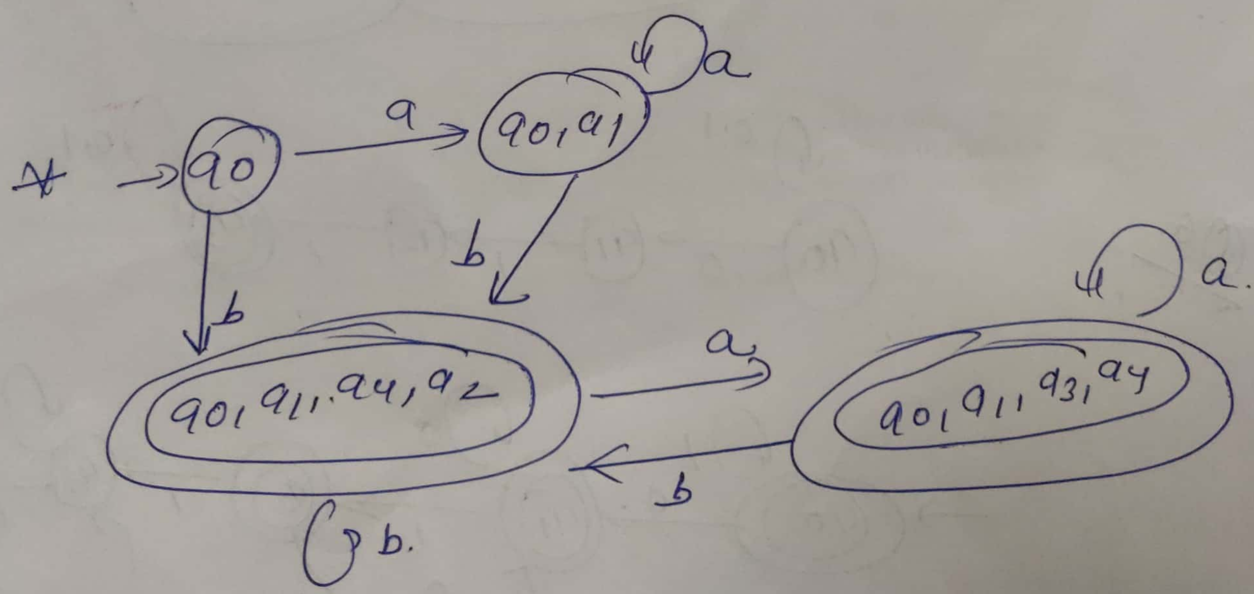




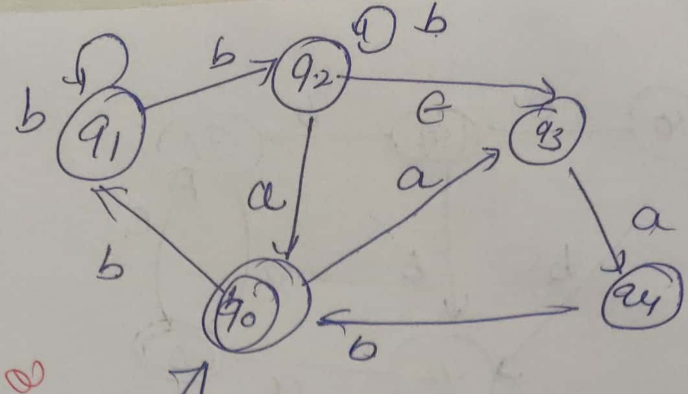
b.G

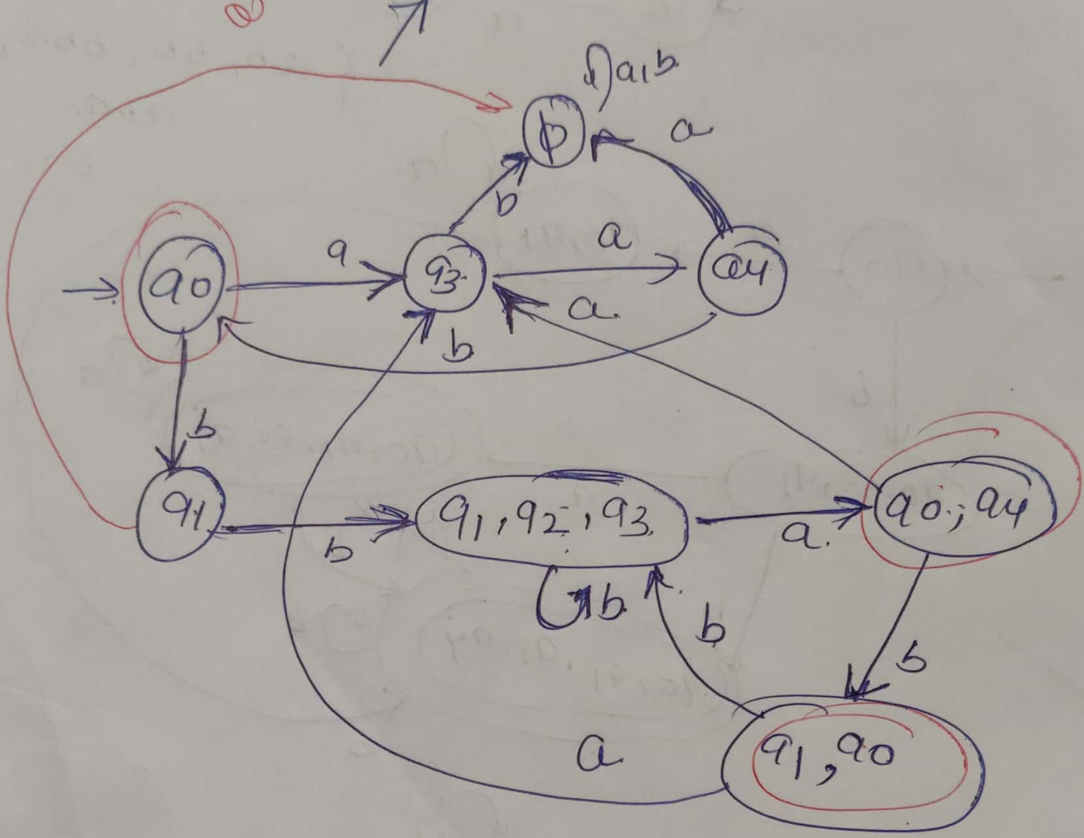
{ab, bb, bba,
aba....}
ba,



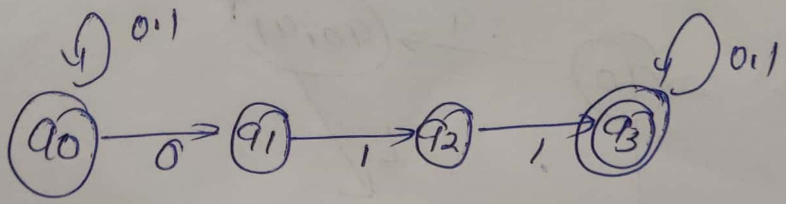


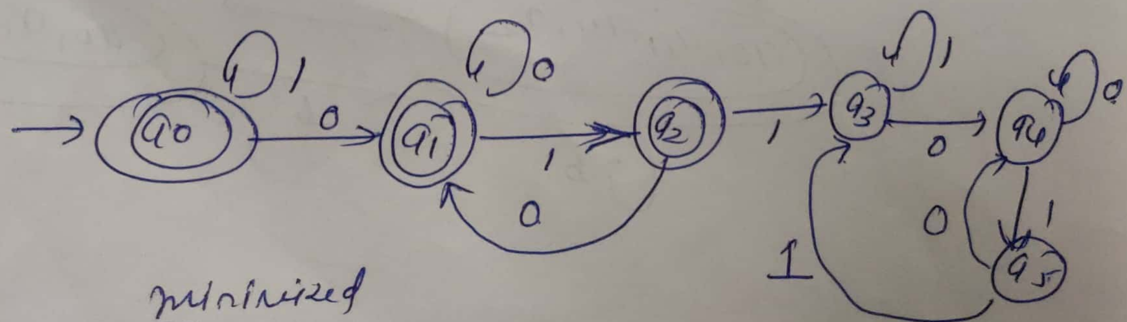
(b)



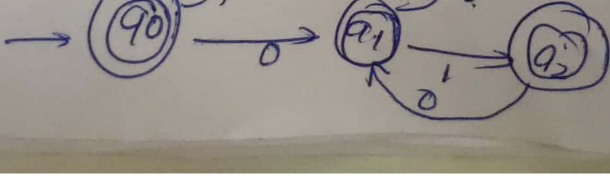


Q6.





minimized

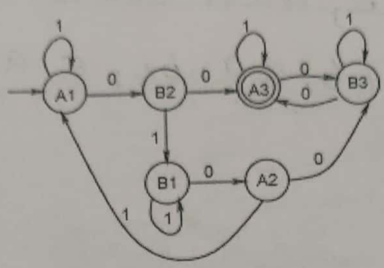


$+a)^* + b$

Q.7)

3. Design a DFA that accepts the language $L_3 = \{x \in \{0,1\}^* : |x|_0$ is even and '00' is a substring of x $\}$. Write down the equations of states and find the regular expression for L_3. [5]

Ans. The DFA is



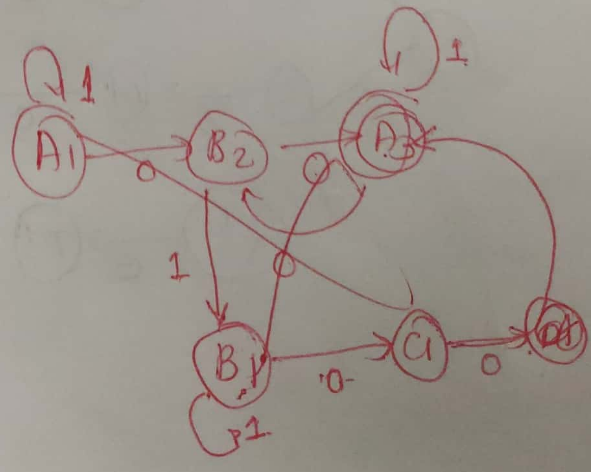
The equations are

$$a_1 = 1a_1 + 0b_2$$

$$\begin{aligned} a_3 &= 1a_3 + 0b_3 + \varepsilon \\ b_1 &= 1b_1 + 0a_2 \\ b_2 &= 0a_3 + 1b_1 \\ b_3 &= 1b_3 + 0a_3 \end{aligned}$$

From $a_3 = 1a_3 + 0b_3 + \varepsilon$ we get $a_3 = 1^*(0b_3 + \varepsilon) = 1^*0b_3 + 1^*$. Again $b_3 = 1b_3 + 0a_3 = 1b_3 + 0(1^*0b_3 + 1^*) = (1 + 01^*0)b_3 + 01^*$. So $b_3 = (1+01^*0)^*01^*$. So $a_3 = 1^*0((1+01^*0)^*01^*) + 1^*$.
$b_1 = 1b_1 + 0a_2$, so $b_1 = 1^*0a_2$.
$a_2 = 0b_3 + 1a_1 = 0(1 + 01^*0)^*01^* + 1a_1$.
$a_1 = 1a_1 + 0b_2 = 1a_1 + 0(0a_3 + 1b_1) = 1a_1 + 00(1^*0((1+01^*0)^*01^*) + 1^*) + 01b_1 = 1a_1 + 00(1^*0((1+01^*0)^*01^*) + 1^*) + 011^*0a_2 = 1a_1 + 00(1^*0((1+01^*0)^*01^*) + 1^*) + 011^*0(0(1+01^*0)^*01^* + 1a_1) = (1 + 011^*01)a_1 + 00(1^*0((1+01^*0)^*01^*) + 1^*) + 011^*00(1+01^*0)^*01^*$
So the final regular expression is

$$a_1 = (1 + 011^*01)^*00(1^*0((1+01^*0)^*01^*) + 1^*) + 011^*00(1+01^*0)^*01^*.$$



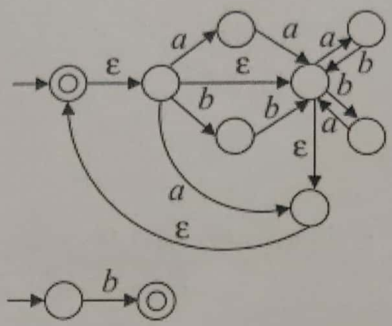
Q8)

1. Construct an ϵ-NFA equivalent to the regular expression $((aa+bb+\epsilon)(ab+ba)^*+a)^*+b$.

~~(b, aaa, bba, a~~

(b, aa, bb, ε, a,

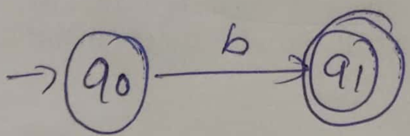
Solution

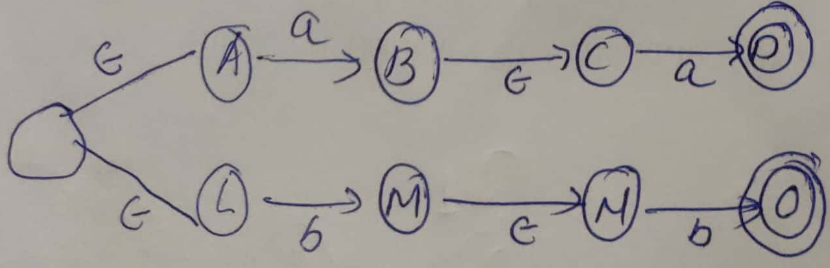


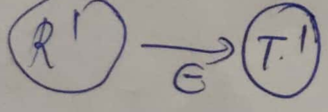
((aa, bb, ε), a)

((aa, bb, ε) (ab +ba)
, a)

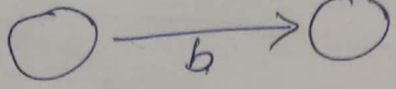
(aa, bb, ε) (aba

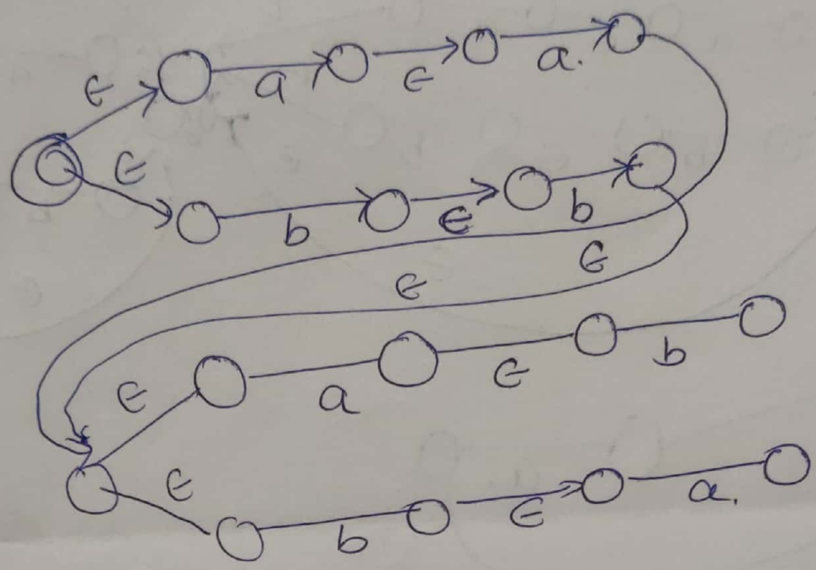


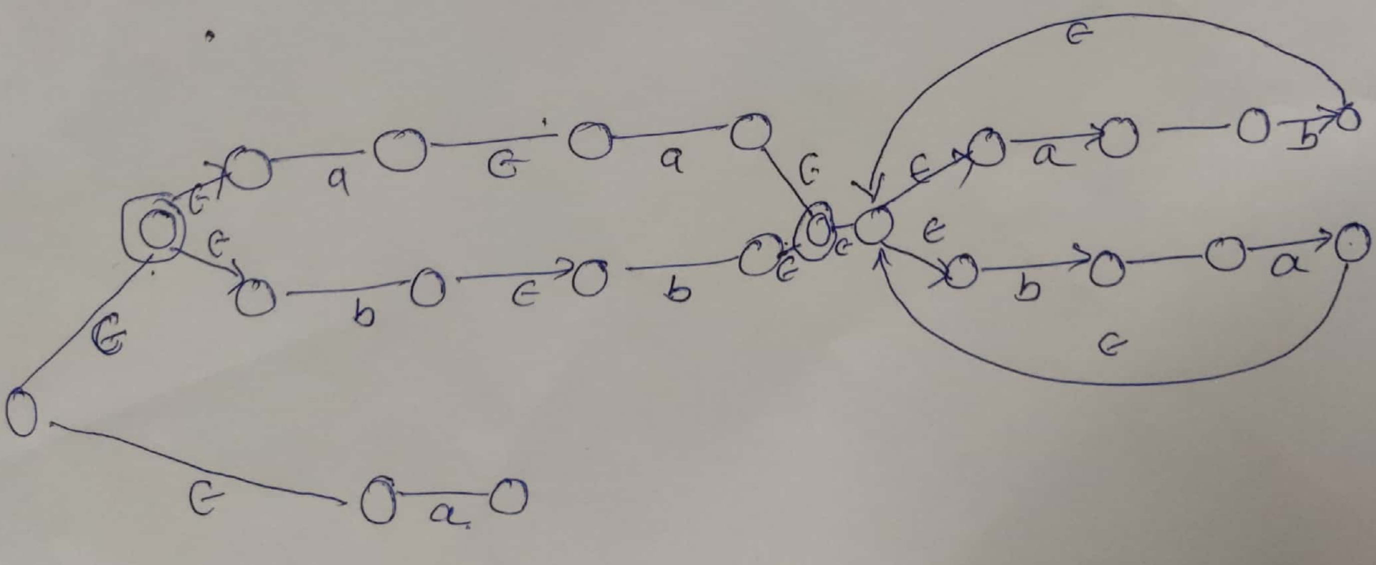




$$((aa+bb+\epsilon)(ab+ba)^* + a)^* + b$$







{ ∈, b, ---- }